

## Pressure recovery in magma due to bubble growth

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[1] Pressure recovery in magma subject to sudden depressurization is investigated in terms of the growth of bubbles containing magmatic volatiles. Assuming conservation of volatile mass and pressure equilibration between the bubbles, melt and a surrounding elastic medium, the final magma pressure is completely determined by its initial pressure and the magnitude of the pressure drop. Simulations show that the initial magma pressure is easily recovered, or even exceeded, when magma containing tiny bubbles is surrounded by a relatively stiff elastic medium under low confining pressure. This result suggests that when processes such as magma withdrawal, dike intrusion, and nearby seismicity decrease the pressure in a magma chamber, magma re-pressurization can occur without new injection of magma. **INDEX TERMS:** 8414 Volcanology: Eruption mechanisms; 8439 Volcanology: Physics and chemistry of magma bodies; 8434 Volcanology: Magma migration; 8499 Volcanology: General or miscellaneous. **Citation:** Nishimura, T. (2004), Pressure recovery in magma due to bubble growth, *Geophys. Res. Lett.*, 31, L12613, doi:10.1029/2004GL019810.

### 1. Introduction

[2] To interpret the spatial and temporal changes in magmatic sources beneath active volcanoes, which have been determined precisely by geodetic and seismological means, it is necessary to first elucidate the forces that control the motion of magma. Buoyancy forces are the most probable candidates for the principal driving force, but may not explain all observed magma migration processes, such as horizontal dike intrusion or intermittent movements. The triggering of long-period seismic tremor has been often interpreted in terms of sudden vesiculation, but few quantitative analyses have been made to date. This study sheds light on the growth of volatile bubbles in magma, which has been studied to understand natural geologic samples, as another important candidate for the forces driving magma movement. By incorporating crustal elasticity into the bubble growth model proposed by Proussevitch *et al.* [1993], in which multiple bubbles of constant radius are closely packed within the melt, we analyze magma chamber re-pressurization in response to sudden depressurization caused. We envisage the likely mechanisms of depressurization to include dike injection, plug opening of a volcanic pipe, and stress changes induced by nearby earthquakes.

[3] The notation and values used for pressure, volume, bubble radius, and other physical parameters are listed at the end of the text and in Table 1. This study uses water for the volatiles in magma.

### 2. Pressure Recovery Due to Bubble Growth

[4] In this model, the magma chamber—either a dike or a volcanic pipe—is embedded in an infinite elastic medium, and the magma itself is treated as melt plus numerous small spherical gas bubbles (Figure 1a). The dike width or pipe radius is assumed to be much larger than the size of each bubble. The melt is a compressible liquid saturated with volatiles, which are presumed to behave as a perfect gas. For simplicity, gravitational and other body forces are neglected. The magma in the chamber and the surrounding elastic medium are initially subject to a uniform confining pressure  $P_s$ .

[5] When the magma chamber is depressurized, in response, for example, to the withdrawal of a proportion of the melt or nearby seismicity, the pressure in the melt decreases within a few tens of seconds as a pressure wave propagates through the chamber. Since the rate of growth of the gas bubbles is controlled by viscosity, diffusivity and pressure differences [e.g., Proussevitch *et al.*, 1993], the gas bubbles maintain their original pressure longer than the enclosing melt (Figure 1b). In this study, therefore, the instantaneous pressure drop,  $\Delta P_0$ , in the magma is presumed to occur in a much shorter time than bubble growth.

[6] Immediately after the magma is depressurized, the pressure of the melt is balanced by the ambient pressure minus the pressure drop:

$$P_{l0} = P_s - \Delta P_0, \quad (1)$$

On the other hand, the gas bubbles maintain their initial pressure:

$$P_{g0} = P_s + 2\sigma/r_{g0}, \quad (2)$$

where  $\sigma$  is the surface tension and  $r_{g0}$  the initial bubble radius. Equations (1) and (2) represent the initial condition for the pressures of the melt and gas bubbles, respectively. With time, the gas bubbles increase in radius due to the pressure gradient built up between the bubbles and ambient melt (Figure 1c). Bubble growth ultimately stops when the pressures of the gas and melt reach an equilibrium expressed as:

$$P_{gf} = P_{lf} + 2\sigma/r_{gf}, \quad (3)$$

Given the necessary conservation of mass, the total mass of volatiles in the magma remains the same throughout the bubble growth stage, in which case:

$$\frac{4}{3}\pi n_g \rho_{gf} r_{gf}^3 = \rho_{l0} (C_0 - C_f) \left(1 - \frac{4}{3}\pi n_g r_{g0}^3\right) + \frac{4}{3}\pi n_g \rho_{g0} r_{g0}^3. \quad (4)$$

**Table 1.** Properties of Magma and the Surrounding Elastic Medium

Property	Symbol	Rhyolite Magma
Henry's constant	$K_H$	$1.6 \times 10^{-11}/\text{Pa}$
Melt density	$\rho_l$	$2200 \text{ kg/m}^3$
Surface tension	$\sigma$	$0.32 \text{ N/m}$
Bulk modulus of melt	$K_l$	$1.38 \times 10^{10} \text{ Pa}$
Temperature	$T$	$1300 \text{ K}$
Gas constant	$R$	$8.31 \text{ J/K/mol}$
Bubble density	$n_g$	$10^8/\text{m}^3$
Molecular weight of water	$M$	$0.018 \text{ kg/mol}$
Elastic rigidity	$\mu$	$1.1 \times 10^{10} \text{ Pa}$
Poisson's ratio	$\nu$	$0.25$

Here it is assumed that no new bubbles are created and that the gas density is negligibly small compared with the melt density. For water dissolved in silicate melt, the volatile concentration can be expressed by Henry's law:

$$C_0 = \sqrt{K_H P_{g0}}, \quad C_f = \sqrt{K_H P_{gf}}, \quad (5)$$

where the gas pressures in the melt are related by

$$\frac{P_{g0}}{\rho_{g0}} = \frac{P_{gf}}{\rho_{gf}} = \frac{RT}{M}. \quad (6)$$

Temperature is assumed to be constant throughout the bubble growth process. Since the melt is compressible, the final melt pressure is related to the melt's volume change via the bulk modulus,  $K_l$ :

$$P_{lf} = P_{l0} - K_l \frac{\left(\frac{V_f}{V_0} - 1\right) - \frac{4}{3}\pi n_g (r_{gf}^3 - r_{g0}^3)}{1 - \frac{4}{3}\pi n_g r_{g0}^3}. \quad (7)$$

The pressure of the melt is balanced by the stress applied by the surrounding elastic medium. When the magma occupies a two-dimensional crack of length  $L$  and volume per unit length  $S_0$ , the initial and final pressures in the melt are related to the deformation of the crack [e.g., Okamura, 1990]:

$$P_{lf} - P_{l0} = \bar{\mu} \left(\frac{V_f}{V_0} - 1\right), \quad (8)$$

where

$$\bar{\mu} = \frac{2\mu S_0}{\pi(1-\nu)L^2}. \quad (9)$$

In the case of a volcanic pipe, this is expressed by:

$$P_{lf} - P_{l0} = 2\mu \left(\sqrt{\frac{V_f}{V_0}} - 1\right) \quad (10)$$

The pressure difference  $P_{lf} - P_{l0}$  represents pressure recovery due to gas bubble growth after an instantaneous pressure drop of  $\Delta P_0$ .

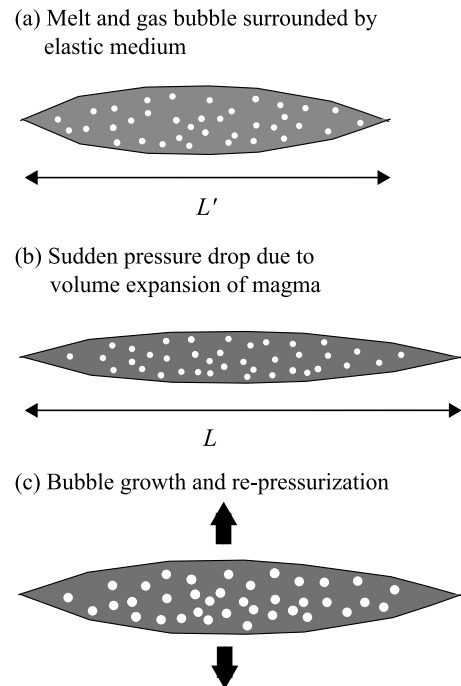
### 3. Simulation Results

[7] Pressure recovery and the final gas bubble radius are calculated by substituting the initial gas bubble radius,

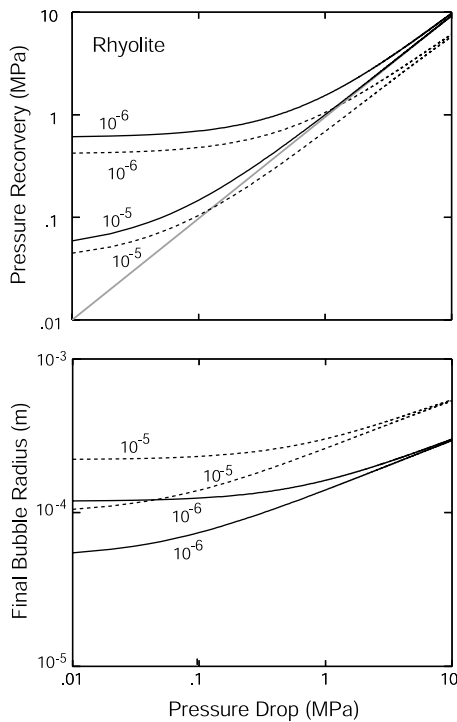
bubble density, pressure drop, and various physical parameters of magma into equations (1)–(10). We use parameters appropriate for rhyolitic magmas as summarized in Table 1, most of which are taken from Table 2 of Proussevitch *et al.* [1993]. Only the results for the case of a crack are shown below, although the basic characteristics of pressure recovery are the same for other magma chamber geometries.

[8] Figures 2 show the relationship of the pressure recovery,  $P_{lf} - P_{l0}$ , and final bubble radius,  $r_{gf}$ , to initial pressure drop,  $\Delta P_0$ , for rhyolitic magma. In our calculations,  $r_{g0} = 10^{-5} - 10^{-6} \text{ m}$ ,  $S_0/L^2 = 0.1 - 0.01$ ,  $P_s = 75 \text{ MPa}$  and  $n_g = 10^8/\text{m}^3$  are assumed. When the pressure drop is small, the pressure drop is sometimes exceeded by the subsequent pressure recovery. This over-pressurization is mainly caused by pressure release from the surface tension of gas bubbles (see equations (2) and (3)). On the other hand, when the pressure drop is large, the magma pressure does not totally recover, probably because the total amount of volatiles dissolved in the melt is insufficient to fill the gas bubbles and attain the initial pressure. We note that the effective rigidity of the elastic medium surrounding the magma chamber,  $\bar{\mu}$ , plays a significant role in the pressure recovery and final bubble radius: as the effective rigidity decreases (i.e., small  $S_0/L^2$  for a crack or small  $\mu$  for a pipe), the pressure recovery also decreases and the final bubble radius increases.

[9] Magma having a large Henry's constant can generally generate a large volume of volatiles (water) from the melt for bubble growth and pressurization. Although Henry's constant for rhyolitic magma is about two times larger than



**Figure 1.** Schematic illustration of an idealized magma chamber and its response to sudden depressurization. (a) Magma containing numerous gas bubbles. (b) Volumetric expansion and pressure drop ( $L > L'$ ). (c) Re-pressurization of the magma due to gas bubble growth.



**Figure 2.** Graph showing pressure recovery and final bubble radius in response to a pressure drop applied to rhyolitic magma in a crack. The solid and dotted lines represent thick ( $S_0/L_0^2 = 0.1$ ) and thin cracks ( $S_0/L_0^2 = 0.001$ ), respectively. The results for two different initial bubble radii ( $10^{-5}$  m and  $10^{-6}$  m) are plotted. In both cases,  $P_s = 75$  MPa and  $n_g = 10^8/\text{m}^3$ . The gray line represents the pressure recovery equal to the pressure drop, and an over-pressurization is expected for the case where the solid and dotted lines are above the gray line.

that for basaltic magma ( $9.0 \times 10^{-12}/\text{Pa}$ ), the difference is too small to cause a significant changes in the pressure recovery. Bubble density has little effect on pressure recovery, although larger final bubble radii are attained when the bubble density is low.

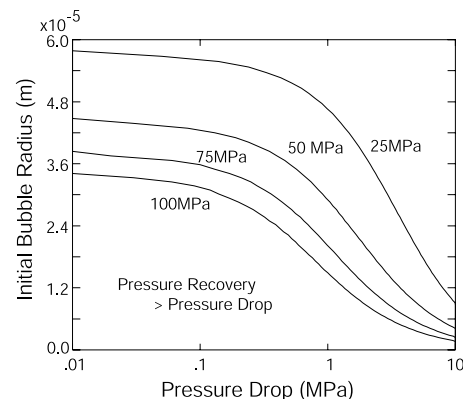
#### 4. Discussion

[10] When the stress intensity factor of a crack exceeds a critical value characteristic of the enclosing medium, the crack extends in length [e.g., *Aki et al.*, 1977] and the total volume of magma increases. Similar volumetric expansion is also expected when excess pressure stored in a magma causes the opening of a plug or volcanic conduit. In both these processes, the magma remaining in the original chamber is ultimately depressurized due to the partial withdrawal of some magma. This implies that when no additional magma is supplied, bubble growth processes are necessary to pressurize magma chamber and to permit further magma migration. The regions beneath each curve in Figure 3 represent initial bubble radius and pressure drop combinations for which the pressure recovery is larger than the triggering pressure drop. For example, in response to a pressure drop of 1 MPa at a confining pressure of 100 MPa,

magmas containing gas bubbles smaller than  $3 \times 10^{-5}$  m in radius will be completely re-pressurized. This highlights the importance of small initial bubble radii in enabling magmas to fully recover from pressure drops at high confining pressure.

[11] This model of pressure recovery associated with bubble growth may be relevant to intermittent dike intrusions observed at active volcanoes [e.g., *Okada and Yamamoto*, 1991; *Fujita et al.*, 2002]. Sudden discharges of large bubbles accumulated beneath the roof of a chamber, which have been interpreted as a trigger for repeated eruptions at Stromboli volcano [e.g., *Ripepe et al.*, 2001] might cause a pressure build-up within the chamber and thereby control eruption dynamics. The triggering of magmatic oscillations that generate VLP and LP seismic events [e.g., *Arciniega-Ceballos et al.*, 1999; *Nishimura et al.*, 2002] may likewise be related to bubble growth in the magma. For example, when a pressure in magma decreases 0.1 MPa, which is likely to occur when 0.001% of magma volume is withdrawn from a thick crack located at a depth of 3000 m (about 75 MPa confining pressure) due to a crack opening or plug opening, over-pressurization of about 0.6 MPa is expected to occur in magma containing gas bubbles with an initial radius of  $10^{-6}$  m and  $n_g = 10^8/\text{m}^3$  (see Figure 3). The time scales for bubble growth processes studied by *Proussevitch et al.* [1993], which vary from less than a few seconds to more than 10 days for basaltic and rhyolitic magmas, are within the range of the observed phenomena. These results suggest that the pressure recovery and increase predicted from the model in this study is enough to cause a new withdrawal of magma and/or to excite volcanic earthquakes and tremor. However, since this model neglects the time factors and gravitational force, it is necessary to include these effects in the model for discussing the observed phenomena more in detail.

[12] Bubble growth in magma also plays an important role on the magma fragmentation and eruption dynamics. For example, a specific value of vesicularity (0.70–0.75)



**Figure 3.** Graph showing the relationship between initial bubble radius and pressure drop satisfying the condition  $P_{if} - P_{i0} = \Delta P_0$  for confining pressures of 25, 50, 75, and 100 MPa. The area below each curve represents combinations of initial bubble radius and pressure drop for which the re-pressurization more than compensates for the pressure drop.

has been often used for magma fragmentation level to understand the explosivity of eruption [e.g., Mader, 1998]. Such fragmentation level is easily obtained when magma ascends in a rigid conduit (i.e., magma is not stressed from the conduit) if no degassing occurs during the ascent. On the other hand, when a magma batch ascends in an elastic conduit, the bubble growth is expected to be suppressed. From this model, the vesicularity in magma is roughly estimated to be less than 0.75 for the surrounding medium with  $\bar{\mu} \geq 4.7 \times 10^6$  Pa, when a magma batch ascends from a depth of 5 km to the ground surface, which corresponds to a depressurization of about 130 MPa. This implies that magma can migrate to the surface without fragmentation even if no gas is escaped from the magma.

[13] The ultimate strain energy produced in the elastic medium is larger than the initial strain energy because of magma chamber re-pressurization. This strain energy is supplied by the thermal energy of the melt. Although the energy equation must be solved in order to analyze the energy balance in detail, the assumption of constant temperature made in this study means that the results would not be significantly different. For example, magma increasing in pressure by 1 MPa within a 10 m radius volcanic pipe produces an increase in elastic strain energy of about  $7 \times 10^4$  J per unit length of the conduit. This amount is approximately  $10^4$  times smaller than the release of thermal energy associated with a 1 K decrease in temperature of a magma having a heat capacity of  $1.3 \times 10^3$  J/kg/K. Such small changes in temperature hardly affect the bubble growth process analyzed in this study.

## 5. Summary

[14] Pressure recovery in a magma subjected to a sudden drop in confining pressure has been modeled by taking into account bubble growth in a compressible melt surrounded by an elastic medium. This model predicts that initial magma pressure is totally recovered or exceeded without any new magma being supplied to the chamber when the initial bubble radius is small, the surrounding elastic medium is rigid, and/or the confining pressure is small.

## Notation

$C$	concentration (kg water/kg melt)
$P$	pressure
$r$	bubble radius
$V$	magma volume

## Subscripts

0	initial condition
f	final condition
l	melt
g	gas

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